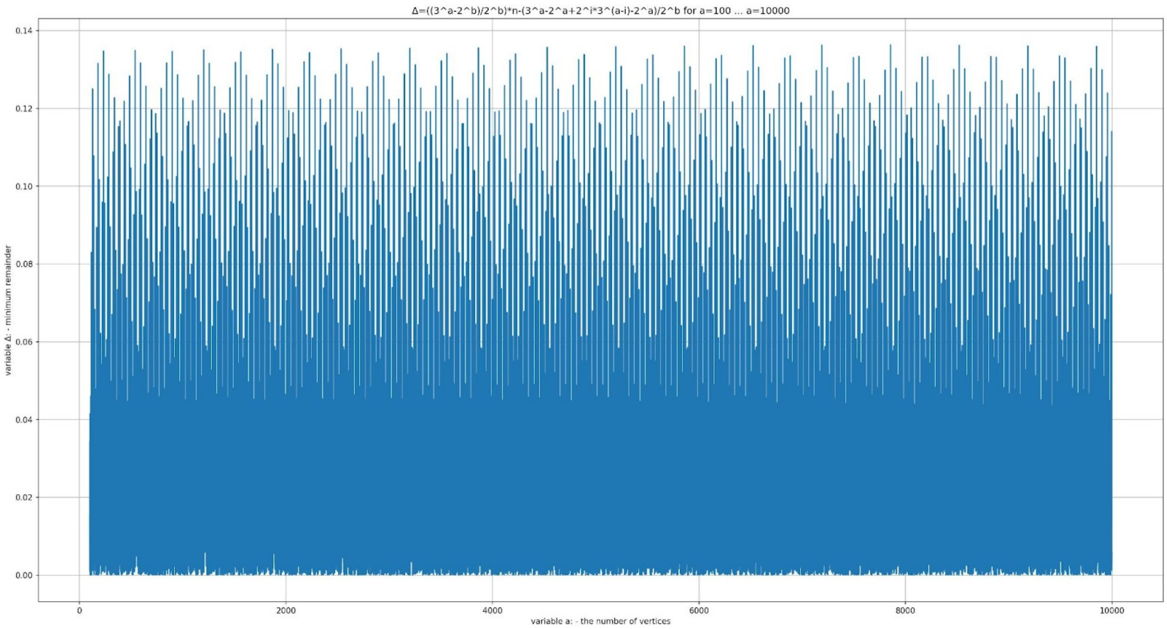
Collatz conjecture - testing

Autor: Ing. Robert Polák (Robopol), Email: robopol@gmail.com, Slovakia

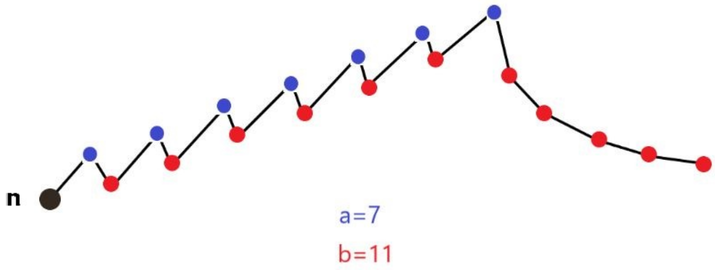


Dátum/date:11.11.2022

Abstract:

This publication directly follows on from the publication - (1) Collatz's conjecture (search for useful relationships) dated 15.8.2022.
The publication tests the relations from (1) and reveals other connections.
Finding evidence for the Collatz hypothesis. ***This work shows that Collatz's hypothesis is apparently true.***

Všeobecný fraktál pre 3n+1/ General fractal for 3n+1



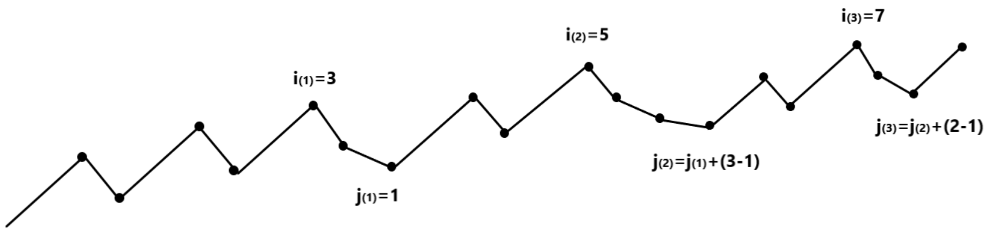
pic. No.1 Význam premenných a, b, n / Meaning of variables a, b, n

V nadväznosti na publikáciu (1) Collatz's conjecture dostaneme tieto rovnice: / In terms of publication (1) Collatz's conjecture, we get the following equations:

$$n = \frac{3^a - 2^a + \varepsilon_k}{2^b - 3^a} \qquad (1.1)$$

$$b = int(\frac{a \ln 3}{\ln 2}) + 1 \qquad (1.2)$$

n − *is the initial number*
a, b − *see pic. No.* 1
ε_k − *is corrective term*, $\varepsilon_k \in$ *integer*



pic. No.2 Význam premenných i_k, j_k / Meaning of variables i_k, j_k

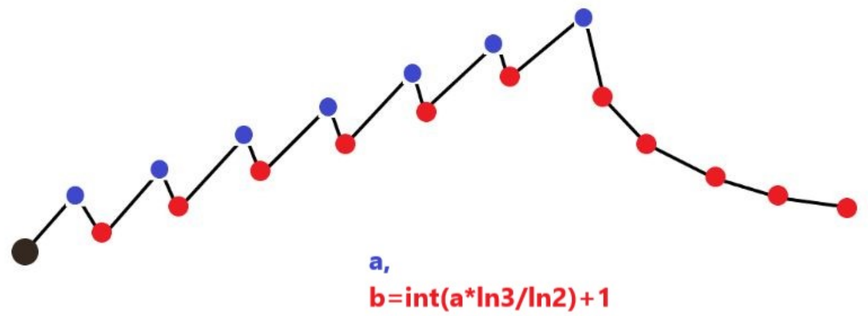
$$\varepsilon_k = \left(2^{i_1}3^{a-i_1} - 2^a\right) + \sum_{i=2}^{<a}\left(2^{j_k} - 2\right)\left(2^{i_k}3^{a-i_k} - 2^a\right) \qquad (1.3)$$

i_k − *peak*, j_k − *decline* : *pic No.* 2

Všeobecný fraktál musí spĺňať kongruenciu (pre platnosť Collatz conjecture): / A general fractal must satisfy the congruence (for the validity of the Collatz conjecture):

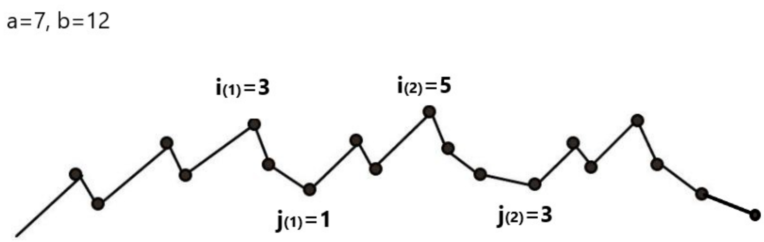
$$3^a - 2^a + \varepsilon_k \equiv 0 \qquad mod\ (2^b - 3^a)$$

Obdobne, v zmysle publikácie (1) dostaneme pre základný fraktál, kde $\varepsilon_k = 0$ platnosť Collatzovej hypotézy na celom definičnom obore (okrem triviálneho riešenia rovnice, kde a=1, b=2).
/ Similarly, in the sense of publication (1), we get for the basic fractal, where $\varepsilon_k = 0$, the validity of the Collatz hypothesis on the domain of a function (except for the trivial solution of the equation, where a=1, b = 2).



pic.No.3 Basic fractal: $\varepsilon_k = 0$

<u>Example:</u>



$$b = int(\frac{7\, ln\, 3}{ln\, 2}) + 1 = 12$$

$$\varepsilon_k = \left(2^{i_1} 3^{a-i_1} - 2^a\right) + \sum_{i=2}^{<a} \left(2^{j_k} - 2\right)\left(2^{i_k} 3^{a-i_k} - 2^a\right) =$$

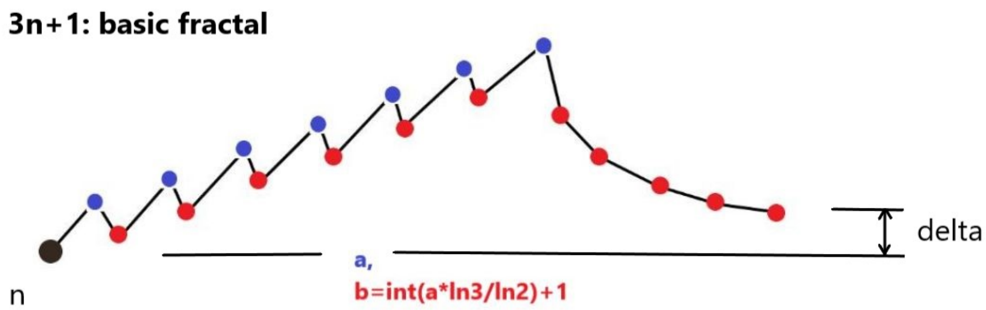
$$= \left(2^3 3^{7-3} - 2^7\right) + \left(2^3 - 2\right)\left(2^5 3^{7-5} - 2^7\right) = 1480$$

$$n = \frac{3^a - 2^a + \varepsilon_k}{2^b - 3^a} = \frac{3^7 - 2^7 + 1480}{2^{12} - 3^7} = 1.8538...$$

$$(3^7 - 2^7 + 1480)\ mod\ (2^{12} - 3^7) = 1630$$

Testovanie v Pythone / Testing in Python

Pre základné testovanie odvodených matematických vzťahov si čitateľ môže stiahnuť zdrojový kód programov v pythone pre tri prípady. Základné veličiny, ktoré program vyráta sú znázornené na obr. 4. Pri testovaní vznikli opakujúce sa vzory s periódou T=665. / For basic testing of the derived mathematical equations, the reader can download the source code of the python programs for the three cases. The basic quantities calculated by the program are shown in fig. 4. During testing, recurring patterns with a period of T=665 were created.

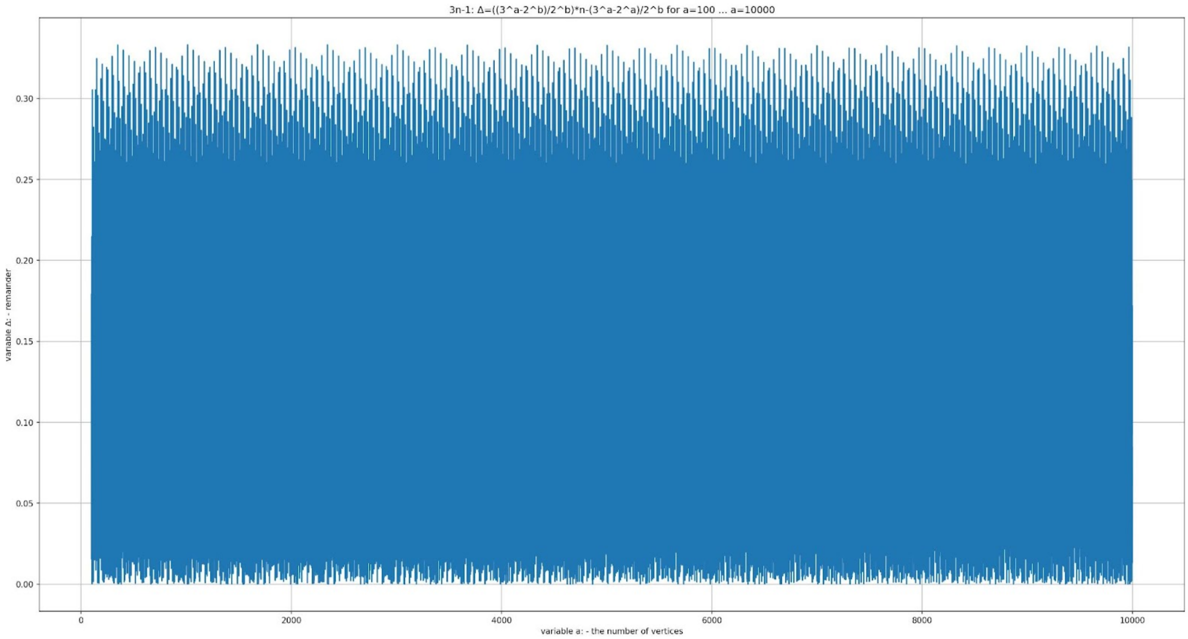


pic. No. 4 Základné veličiny / Basic variables.

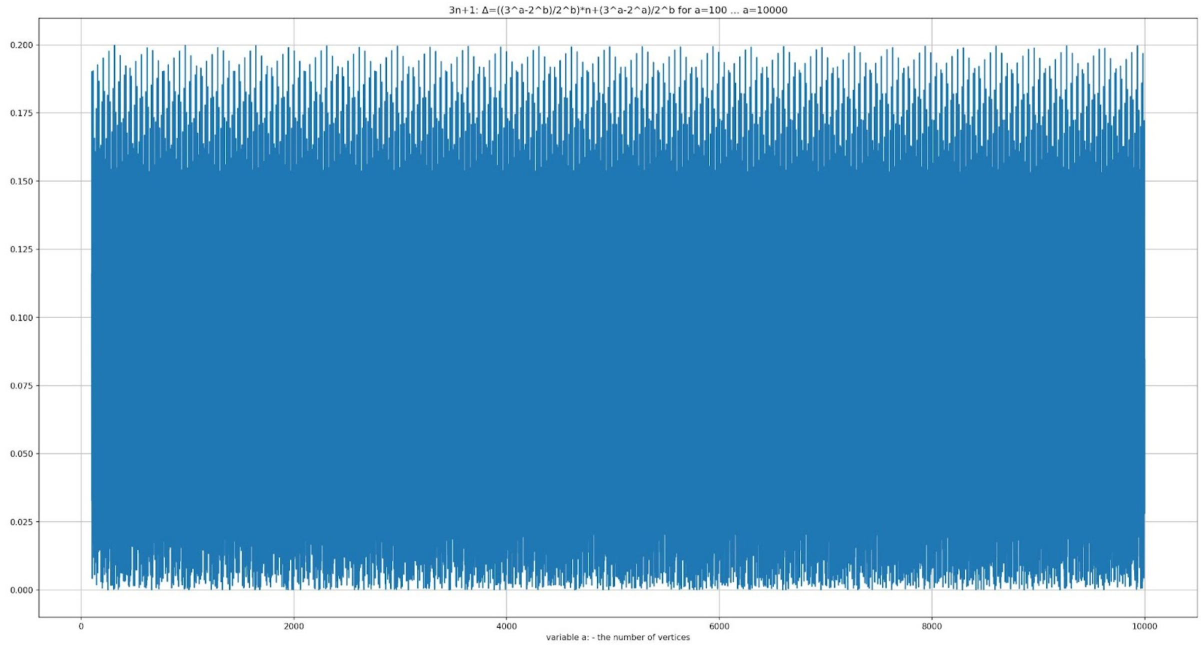
Basic Fractal Test

```
****************************************************************************************
Basic fractal -Collatz conjecture : 3n-1, 3n+1

Author: Ing. Robert Polak
Contact Info: robopol@robopol.sk
website: https://www.robopol.sk
Purpose:
        Collatz hypothesis test 3n-1, for basic fractal,
        n=(3^a-2^a)/(3^a-2^b)
        or
        Collatz hypothesis test 3n+1, for basic fractal,
        n=(3^a-2^a)/(2^b-3^a)

type: console program
Copyright notice: This code is Open Source and should remain so.
To end the program, press 0 and the enter.
****************************************************************************************

Choose enter 1 for 3n-1 or 2 for 3n+1:
▯
```

download source in GitHub: [basic_fractal_test_collatz.py]



pic.No.5: Delta graph for sequence 3n-1



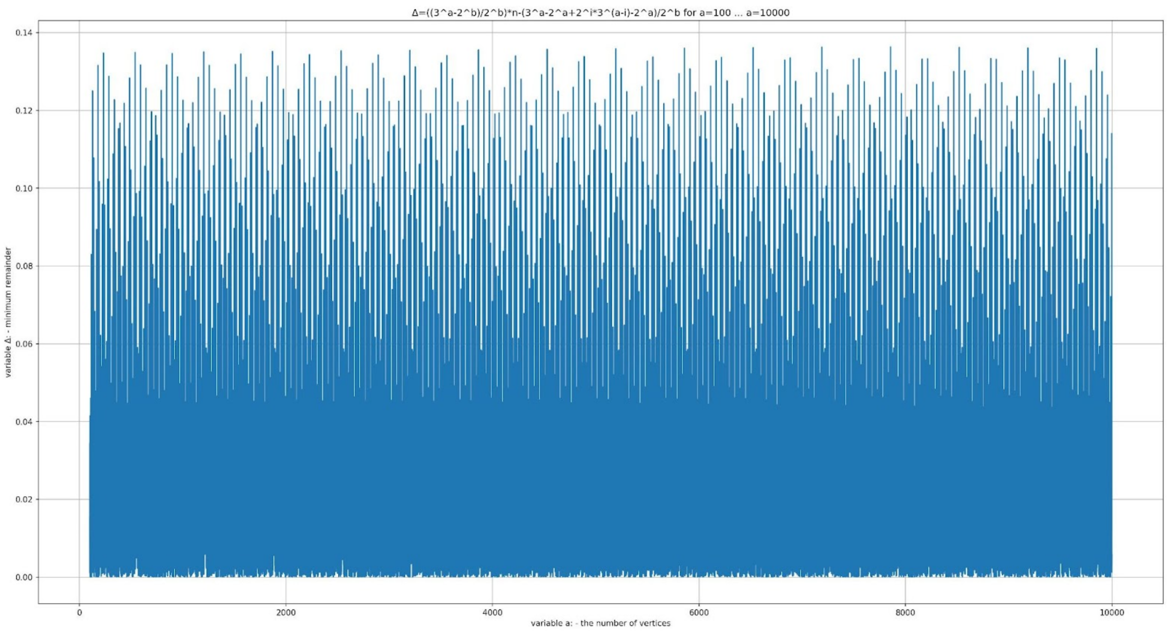
pic.No.6: Delta graph for sequence 3n+1

Z priebehu vidíme opakujúce sa sekvencie. Program vypíše hodnoty delta, aj rozdiely (difference) medzi hodnotami posunutými o periódu T=665. / From the course we see repeating sequences. The program outputs the delta values as well as the differences (differences) between the values shifted by the period T=665.

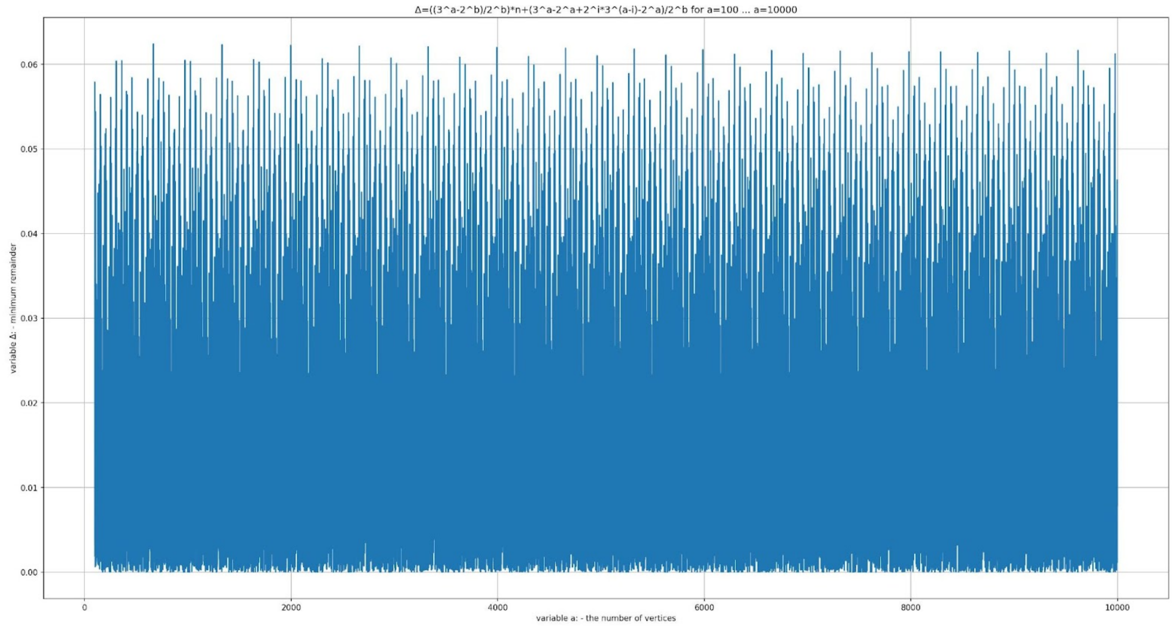
Simple test Collatz conjecture

```
*****************************************************************************************
Simple test Collatz conjecture : 3n-1, 3n+1

Author: Ing. Robert Polak
Contact Info: robopol@robopol.sk
website: https://www.robopol.sk
Purpose:
        Collatz hypothesis test 3n-1, 3n+1 for n-fractal,
        only selected combinations, ε(k)=2^i*3^(a-i)-2^a

type: console program
Copyright notice: This code is Open Source and should remain so.
To end the program, press 0 and the enter.
*****************************************************************************************

Choose enter 1 for 3n-1 or 2 for 3n+1:
▯
```

download source in GitHub: simple_test_collatz.py



pic.No.7: Delta graph for sequence 3n-1



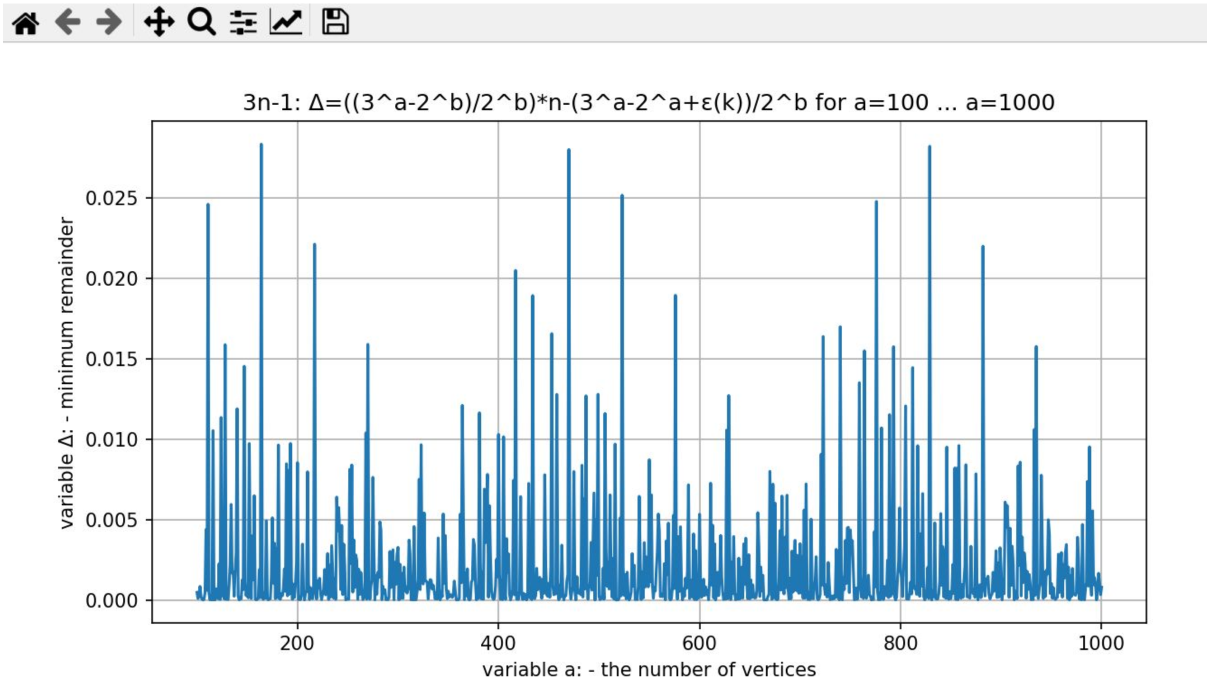
pic.No.8: Delta graph for sequence 3n+1

Z priebehu vidíme opakujúce sa sekvencie. Program vypíše hodnoty delta, aj rozdiely (difference) medzi hodnotami posunutými o periódu T=665. / From the course we see repeating sequences. The program outputs the delta values as well as the differences (differences) between the values shifted by the period T=665.

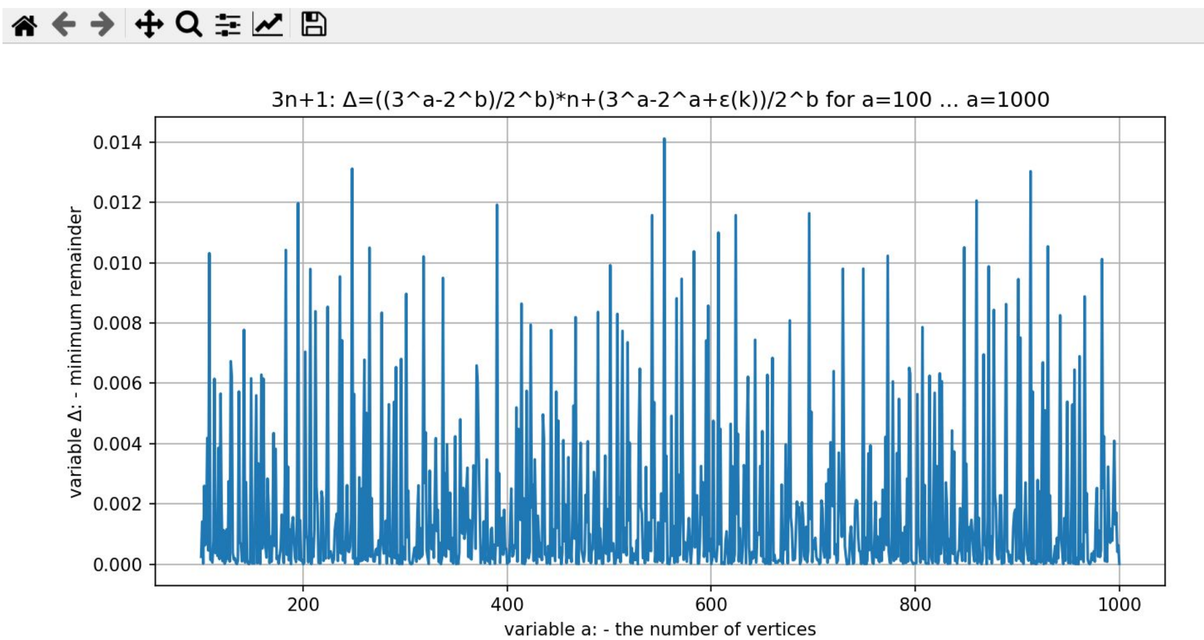
Advanced test Collatz conjecture

```
************************************************************************************
Advanced test Collatz conjecture(2): 3n-1, 3n+1

Author: Ing. Robert Polak
Contact Info: robopol@robopol.sk
website: https://www.robopol.sk
Purpose:
        Collatz hypothesis test 3n-1, 3n+1 for n-fractal,
        only selected combinations, ε(k)=2^i*3^(a-i)-2^a+2*(2^j*3^(a-j)-2^a)

type: console program
Copyright notice: This code is Open Source and should remain so.
To end the program, press 0 and the enter.
************************************************************************************

Choose enter 1 for 3n-1 or 2 for 3n+1:
█
```

download source in GitHub: [advanced_test_collatz.py]



pic.No.9: Delta graph for sequence 3n-1



pic.No.10: Delta graph for sequence 3n+1

From the course we see repeating sequences. The program outputs the delta values as well as the differences (differences) between the values shifted by the period T=665.

Príklad hodnôt delta a rozdielov pre jednotlivé testy / Example of delta values and differences for individual tests.

	A	B	C	D	E	F
1						
2	**3n+1**	period	T=665	test basic fractal		diference
3	1332	0,125098226036732	1997	0,125147342271170		0,000049116234437
4	1333	0,062942017165296	1998	0,063163040220264		0,000221023054969
5	1334	0,101396743563014	1999	0,10131385991740 1		-0,000082883645613
6	1335	0,013967435630142	2000	0,013138599174009		-0,000828836456132
7	...	...	...	...		...
8	**3n-1**	period	T=665	test basic fractal		diference
9	540	0,159943087174453	1205	0,159862759373185		-0,000080327801268
10	541	0,140128053857481	1206	0,140308791410335		0,000180737552854
11	542	0,015928390466736	1207	0,017238737724928		0,001310347258193
12	543	0,105096040393110	1208	0,105231593557751		0,000135553164641
13	...	...	...	...		...
14	**3n+1**	period	T=665	simple test		diference
15	1332	0,062330832936739	1997	0,062246243866319		-0,000084589070420
16	1333	0,000611184228557	1998	0,000916796353946		0,000305612125389
17	1334	0,018056134226960	1999	0,017969612341759		-0,000086521885201
18	1335	0,002357424881576	2000	0,002163225007670		-0,000194199873907
19	...	...	...	...		...
20	**3n-1**	period	T=665	simple test		diference
21	540	0,134911777406487	1205	0,135048632179019		0,000136854772531
22	541	0,018972043136457	1206	0,019147491615977		0,000175448479520
23	542	0,002020648158601	1207	0,000711084466829		-0,001309563691772
24	543	0,014229032352343	1208	0,014360618711983		0,000131586359640
25	...	...	...	...		...

Tab. No.1 Example of delta values and differences for individual tests.

Perióda T=665 / Period T=665

Príčinou periódy v testoch, viď. (obr.7-10) je táto skutočnosť: / The reason for the repeating patterns (see pic. No 7-10) is:

$$665\ \frac{ln\,3}{ln2} \sim int\left(665\ \frac{ln\,3}{ln2}\right) \qquad (2.0)$$

$$665\ \frac{ln\,3}{ln2} - int\left(665\ \frac{ln\,3}{ln2}\right) = 0.0000629795... \qquad (2.1)$$

pretože/ because
v rovnici pre premennú b sa zaokrúhľuje na celé čísla / in the equation for variable b is rounded to integer numbers.
for 3n-1:

$$b = int(\frac{a\,ln\,3}{ln\,2})$$

for 3n+1:

$$b = int(\frac{a\,ln\,3}{ln\,2}) + 1$$

Vyhodnotenie testov / Evaluation of tests

Testy rovnice (1.3) ukazujú, že sa opakujú vzory s maličkými rozdielmi pre delta. Teda intuitívne sa javí, že Collatzová hypotéza by mala platiť do nekonečna. Nie je to však exaktný dôkaz. Triviálne riešenie (príklad 1,4,2,1) je skôr výnimka ako pravidlo. / Tests of equation (1.3) show that there are repeating patterns with small differences for delta. Thus, it appears intuitively that the Collatz hypothesis should have been valid indefinitely. However, it is not an exact proof. A trivial solution (example 1,4,2,1) is the exception rather than the rule.

Pokus o zjednodušenie rovnice 1.3 / Attempt to simplify equation 1.3

$$\varepsilon_k = \left(2^{i_1}3^{a-i_1} - 2^a\right) + \sum_{i=2,\, j=2}^{i,j<a}\left(2^{j_k} - 2\right)\left(2^{i_k}3^{a-i_k} - 2^a\right)$$

$$- 2^a + \sum_{i=2}^{<a}\left(2^{j_k} - 2\right)\left(- 2^a\right) =- 2^a\left(2^{j_{end}+1} - 2j_{end} - 1\right) \qquad (3.0)$$

$$\varepsilon_k = 2^{i_1}3^{a-i_1} + \sum_{i=2,\, j=2}^{i,j<a}\left(2^{j_k} - 2\right)2^{i_k}3^{a-i_k} - 2^a\left(2^{j_{end}+1} - 2j_{end} - 1\right) \qquad (3.1)$$

for 3n-1:

$$n = \frac{3^a - 2^a + \varepsilon_k}{3^a - 2^b} \Rightarrow \varepsilon_k = (n - 1)3^a - n2^b + 2^a$$

if is Collatz (3n-1) true must be: (except for trivial solutions)

$$(n - 1)3^a - n2^b + 2^a \neq 2^{i_1}3^{a-i_1} + \sum_{i=2,\, j=2}^{i,j<a}\left(2^{j_k} - 2\right)2^{i_k}3^{a-i_k} - 2^a\left(2^{j_{end}+1} - 2j_{end} - 1\right)$$

$$(n - 1)3^a - n2^b + 2^a\left(2^{j_{end}+1} - 2j_{end}\right) \neq 2^{i_1}3^{a-i_1} + \sum_{i=2,\, j=2}^{i,j<a}\left(2^{j_k} - 2\right)2^{i_k}3^{a-i_k}$$

(3.2)

for 3n+1:

$$n = \frac{3^a - 2^a + \varepsilon_k}{2^b - 3^a} \Rightarrow \varepsilon_k = n2^b - (n + 1)3^a + 2^a$$

if is Collatz (3n+1) true must be:

$$n2^b - (n + 1)3^a + 2^a \neq 2^{i_1}3^{a-i_1} + \sum_{i=2,\, j=2}^{i,j<a}\left(2^{j_k} - 2\right)2^{i_k}3^{a-i_k} - 2^a\left(2^{j_{end}+1} - 2j_{end} - 1\right)$$

$$n2^b - (n + 1)3^a + 2^a\left(2^{j_{end}+1} - 2j_{end}\right) \neq 2^{i_1}3^{a-i_1} + \sum_{i=2,\, j=2}^{i,j<a}\left(2^{j_k} - 2\right)2^{i_k}3^{a-i_k}$$

(3.3)

Problematický je člen $\sum_{i=2,\, j=2}^{i,j<a} \left(2^{j_k} - 2\right) 2^{i_k} 3^{a-i_k}$, ktorý nie je možné ďalej zjednodušiť. / The equation $\sum_{i=2,\, j=2}^{i,j<a} \left(2^{j_k} - 2\right) 2^{i_k} 3^{a-i_k}$ is problematic.

Basic fractal for $3n \pm 0$

V zmysle obr.4 vytvoríme rovnicu pre základný fraktál ($3n \pm 0$) a dostaneme: / In accordance with Fig. 4, we create an equation for the basic fractal (3n0) and get:

for $3n \pm 0$:

$$\Delta = \left| \frac{3^a}{2^b} n - n \right| \tag{4.0}$$

z rovnice plynie, že pre n>0 je vždy Δ>0 / it follows from the equation that for n>0 it is always >0.

Rovnica (4.0) dokonca vyhovuje všeobecnému fraktálu. Teda nezáleží na kombináciách. Všeobecný fraktál sa počíta rovnako ako základný fraktál, čo pre postupnosť $3n \pm 1$ neplatí. / Equation (4.0) even fits the general fractal. So the combinations don't matter. The general fractal is calculated in the same way as the basic fractal, which is not the case for the $3n \pm 1$ sequence.

Vyjadríme Δ pre základný fraktál 3n+1 / Δ for the base fractal 3n+1 is:

for $3n + 1$:

$$\Delta = \left| \frac{3^a - 2^b}{2^b} n + \frac{3^a - 2^a}{2^b} \right| \tag{4.1}$$

<u>example:</u>

$a = 657,\ b = int(\frac{657\, ln\, 3}{ln\, 2}) + 1 = 1042$

for 3n+1:

$$n = int\left(\frac{3^{657} - 2^{657} + 0}{2^{1042} - 3^{657}} \right) = 2$$

$$\Delta = \left| \frac{3^{657} - 2^{1042}}{2^{1042}} n + \frac{3^{657} - 2^{657}}{2^{1042}} \right| = 0.12703..$$

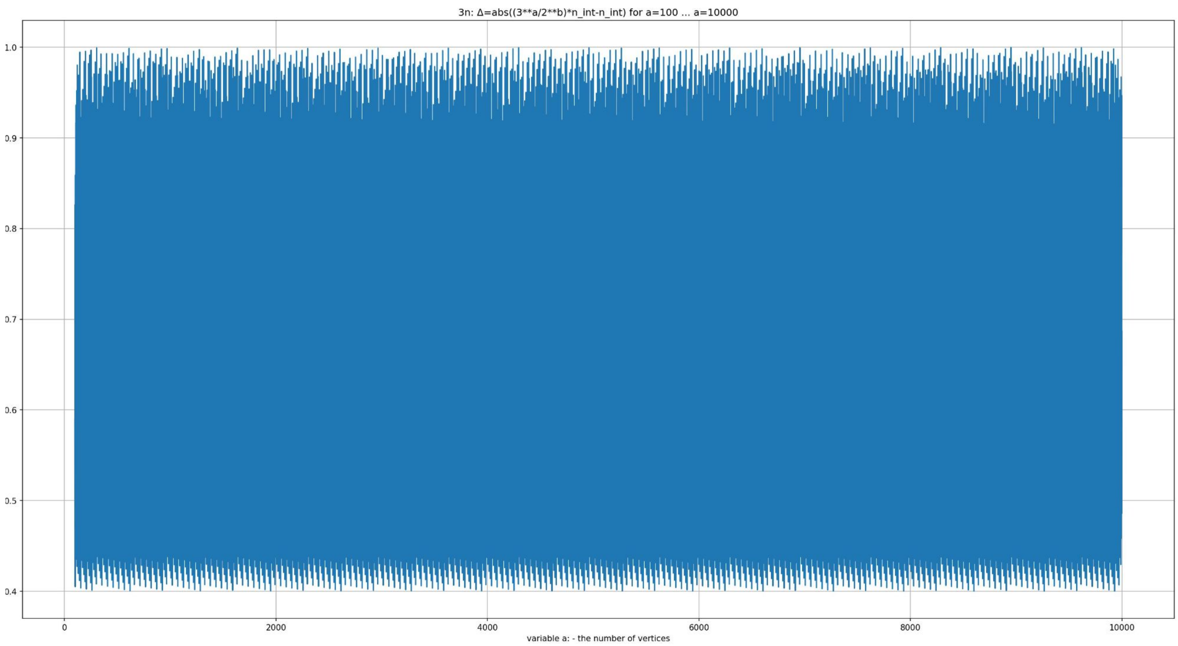
for 3n:

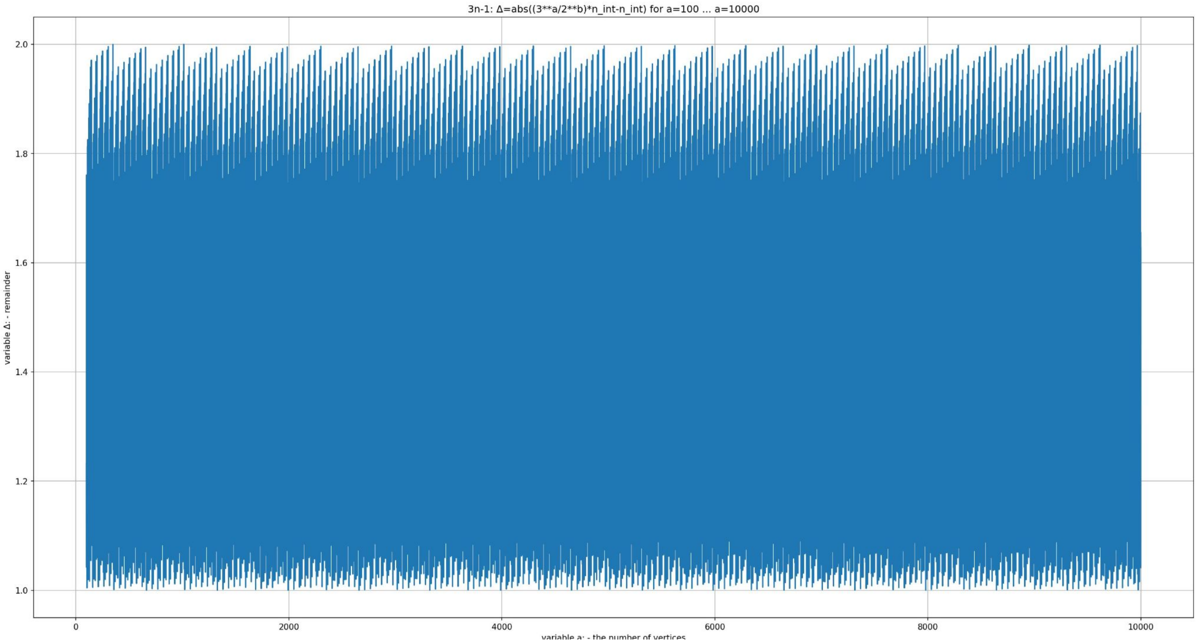
$$\Delta = \left| \frac{3^{657}}{2^{1042}} 2 - 2 \right| = 0.7513..$$

3n - test Collatz conjecture

```
*******************************************************************************************
Collatz conjecture : 3n

Author: Ing. Robert Polak
Contact Info: robopol@robopol.sk
website: https://www.robopol.sk
Purpose:
        Collatz hypothesis test 3n,
        n_int=int((3**a-2**a)/(2**b-3**a))

        delta=abs((3**a/2**b)*n_int-n_int)

type: console program
Copyright notice: This code is Open Source and should remain so.
To end the program, press 0 and the enter.
*******************************************************************************************

Choose enter 1 for 3n-1 or 2 for 3n+1:
```

download source in GitHub: [3n_test_collatz.py]()





pic.No.11: Delta graph for sequence 3n

Testovanie ukazuje podobnosť s priebehom obr.6. Rozdiel je v hodnotách, kde pre postupnosť 3n má delta o niečo vyššie hodnoty. Taktiež je tu prítomná perióda 665. / The testing shows a similarity with the course in Fig.6. The difference is in the values, where for the sequence 3n the delta has higher values. The 665 period is also present.

Záver/ conclusion:

Postupnosť $3n \pm 1$ sú si veľmi podobné v správaní a priebehu s postupnosťou 3n. To je však očakávaný jav, pretože pre veľké čísla $n \to \infty, \ \pm 1$ zohráva čoraz menšiu rolu. Platí teda, že $3n \pm 1 \sim 3n.$ / The $3n \pm 1$ sequence is very similar in behavior and progression to the 3n sequence. However, this is an expected phenomenon, because for large numbers $n \to \infty, \ \pm 1$ plays an increasingly smaller role. So it is true that $3n \pm 1 \sim 3n.$

Pre postupnosť 3n však vieme, že pre žiadne číslo - n nedosiahne vývoj rovnakú úroveň. Nedôjde teda nikdy k opakujúcej sa sekvencii čísiel. Neexistuje ani triviálne riešenie. Pre všetky kombinácie ***platí rovnaká rovnica 4.0*** / However, for the sequence 3n, we know that we will not achieve the same level of development for any number - n. So there will never be a repeating sequence of numbers. There is not even a trivial solution. The same ***equation 4.0 applies to all combinations***.

Jednotlivé testy naznačujú, že Collatzova hypotéza je správna a výnimky najmä v postupnosti 3n-1 platia iba pre malé čísla n. Pre malé čísla existujú triviálne riešenia, kde +-1 v rovnici vykompenzuje postupnosť 3n, ktorá nikdy nedosiahne opakujúcu sa sekvenciu čísiel.
/ individual tests indicate that Collatz's hypothesis is correct and the exceptions, especially in the sequence 3n-1, apply only to small numbers n. For small numbers,

there are trivial solutions where the +-1 in the equation compensates for the sequence of 3n, which never reaches a repeating sequence of numbers.

Z hľadiska rastu postupnosti $3n \pm 1$ nad všetky medze nie je v tejto práci rozoberaná, avšak z hľadiska pravdepodobnosti je vylúčené, aby nejaká postupnosť $3n \pm 1$ časom nenarazila na hodnotu 2^k, $for\ k \in N$.
/ From the point of view of the growth of the $3n \pm 1$ sequence beyond all limits, it is not discussed in this work, but from the point of view of probability, it is excluded that the $3n \pm 1$ sequence does not encounter the value 2^k, $for\ k \in N$.